Indian Statistical Institute, Bangalore

M. Math II-JRF Math I

First Semester - Topology III

Final Exam Duration: 3 hours Date : Nov 11, 2016

Max Marks: 50

(1) Compute the cohomology of $\mathbb{S}^2 \times \mathbb{S}^1$ using Kunneth formula and Universal coefficient theorem. Does it match with $H^*(\mathbb{S}^2;\mathbb{Z}) \otimes H^*(\mathbb{S}^1;\mathbb{Z})$. (5+5 = 10 marks)

(2) For $G \in \{\mathbb{Z}, \mathbb{Z}_2\}$, Compute the ring structure for the cohomology with G-coefficients of the klein bottle. (5+5=10 marks)

(3) Show that $\mathbb{C}\mathbf{P}^2$ and $\mathbb{S}^4 \vee \mathbb{S}^2$ have the same homology and cohomology groups but are not homotopy equivalent. (5+5=10 marks)

(4) If M is a compact oriented manifold of dimension n, with finitely generated free homology groups, then the (unreduced) suspension ΣM can not be a manifold unless M has the same homologies as a sphere. (8+2=10 marks)

(5) (a) Compute the ring structure of $H^*(\mathbb{R}\mathbf{P}^n;\mathbb{Z}_2)$. (2 marks)

(b) Using the cup product structure, show that there is no map $\mathbb{R}\mathbf{P}^n \mapsto \mathbb{R}\mathbf{P}^m$ inducing a non trivial map $H^1(\mathbb{R}\mathbf{P}^m;\mathbb{Z}_2) \mapsto H^1(\mathbb{R}\mathbf{P}^n;\mathbb{Z}_2)$ if $n > m$. What is the corresponding result for maps $\mathbb{C}\mathbf{P}^n \mapsto \mathbb{C}\mathbf{P}^m$?. (3 marks)

(6) (a) Show that $\mathbb{C}\mathbf{P}^\infty = K(\mathbb{Z};2)$. (2 marks)

(b) Show that $\pi_2(\mathbb{S}^1 \vee \mathbb{S}^2)$ is infinitely generated. (3 marks)